THE ACCELERATION OF ELECTRONS IN A BOUNDARY LAYER

Yu. S. Sigov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 3, pp. 25–30, 1967

The effect of the redistribution of energy between ion and electron components for the motion of a plasma in a nonuniform magnetic field is considered on the example of a flat model of an equilibrium boundary layer between a rarefied plasma and a magnetic field in the relativistic invariant form. The relativistic and polarization corrections to the classical theory are found. Results are given for a numerical solution of the problem.

An interesting aspect of the problem of the reflection of a plasma from a "magnetic wall" is the question of the redistribution of energy between heavy and light components.

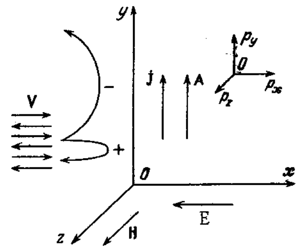


Fig. 1

Chapman and Ferraro [2], in connection with the problem of the interaction of the solar corpuscular stream with the geomagnetic field, have considered the problem of the normal incidence of a nonrelativistic monoenergetic plasma stream on a uniform magnetic field. They have shown that if there is strictly electrostatic coupling between the electrons and the

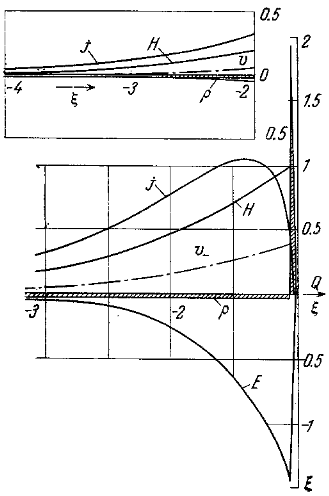


Fig. 2

ions (i.e., assuming that the plasma is neutral at every point) then they exchange energies at the point of reflection.

$$^1/_2\, m_- v_{(-)}^2 = {}^1/_2 m_+ U^2 = {}^1/_2 m_- U^2 + m_+ U^2 (1 - \mu),$$
$$^1/_2 m_+ v_{(+)}^2 = {}^1/_2 m_- U^2 = {}^1/_2 m_+ U^2 \mu$$
$$(u_{0+} = u_{0-} = U,\ v_0 = w_0 = 0,\ \mu = m_- / m_+)\,. \quad (1)$$

Here m is the mass and V = {u, v, w} is the velocity of a particle, the plus and minus subscripts refer to the ion and electron components, respectively, the subscript 0 denotes the velocity at $-\infty$ relative to the "magnetic wall," and enclosing a subscript in parenthesis indicates that the corresponding quantity refers to the reflection point of a particle, where $u_{(\pm)} = 0$.

The spatial arrangement of the boundary layer and the direction of the principal vector quantities are given in Fig. 1. The plasma stream incident on the magnetic field H_0 with some particle distribution function which is specified for $x \to -\infty$, where H = rot and A = 0, is completely reflected from the "magnetic wall." The current j which arises in the plane of the boundary in this case ensures the complete screening of the plasma from the magnetic field, the pressure of which is balanced by the dynamic plasma pressure. Since the heavy ions, with a greater initial momentum than the electrons, have a tendency to penetrate more deeply into the magnetic field, in general there is a charge separation which leads to the appearance of an electro-

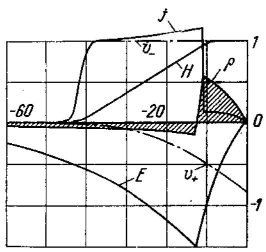


Fig. 3

static field $E = -\nabla\Phi$. The somewhat unusual form of the approximate trajectories of ions and electrons in the transition magnetic field (Fig. 1) is explained by the

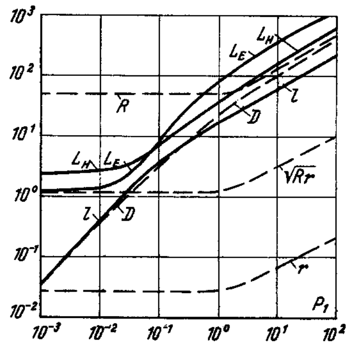


Fig. 4

electrostatic forces coupling particles with opposite charges, so that the acceleration of electrons in the field E is accompanied by the simultaneous deceleration of the ions. The entire pattern is completely self-consistent: the particles motion occurs in an electromagnetic field which in turn is completely specified by the state of the plasma.

In 1958, V. I. Veksler [3] drew attention to the universal nature of this effect, pointing out that pumping of energy from the heavy to the light component, leading to the light component becoming relativistic, can occur for any displacement of a neutral plasma in a nonuniform magnetic field if the electrons and ions exhibit some initial spread of velocities. The "transverse" heating of the electron component was subsequently treated in [4], where the problem of the magnetized-plasma motion in a nonuniform magnetic field was solved in the drift approximation, and in [5], in which the Veksler effect was considered in the single-particle approximation on the example of the plane Chapman-Ferraro model, and in which the question of the polarization corrections to formula (1) was raised.

The present paper is based largely on the results of [1], in which the spatial structure of the boundary layer was obtained by means of a numerical solution of the complete system of Vlasov equations, describing the self-consistent electromagnetic field in a Chapman-Ferraro layer

$$\frac{d^2\Phi}{dx^2} = -4\pi\langle\rho\rangle =$$

$$= 4\pi e\left\{\int f_-(x, \mathbf{p}^\circ)\, dp_x^\circ dp_y^\circ - \int f_+(x, \mathbf{p}^\circ)\, dp_x^\circ dp_y^\circ\right\}, \qquad (2)$$

$$\frac{d^2A}{dx^2} = -\frac{4\pi}{c}\langle j\rangle =$$

$$= 4\pi ce\left\{\int \frac{p_y^\circ + eA/c}{E_-^\circ} f_-(x, \mathbf{p}^\circ)\, dp_x^\circ dp_y^\circ - \right.$$

$$\left. - \int \frac{p_y^\circ - eA/c}{E_+^\circ} f_+(x, \mathbf{p}^\circ)\, dp_x^\circ dp_y^\circ\right\}, \qquad (3)$$

for the boundary conditions

$$\Phi(-\infty) = A(-\infty) = 0,\ E(\infty) = \Phi'(\infty) = 0,$$

$$H(\infty) = A'(\infty) = H_0 = (8\pi\ \langle p_{xx}\rangle)^{1/2}. \qquad (4)$$

Here $\varepsilon_\pm = \pm e$, u_k is the four-velocity of a particle, p_x and p_y are the x and y components of the four-momentum $p_k = mcu_k = p_k^\circ - \varepsilon A_k/c$. Let us recall that the particle distribution functions in an unperturbed plasma as $x \to -\infty$ were taken in the form

$$f_\pm(p_{x0}, p_{y0}) = \frac{n_0}{2|P_{1\pm}|}\delta\left(\frac{p_{y0}}{m_\pm c}\right), \qquad \frac{p^2_{x0}}{m_\pm^2 c^2} \leqslant P^2_{1\pm},$$

$$f_\pm(p_{x0}, p_{y0}) = 0, \qquad \frac{p^2_{x0}}{m_\pm^2 c^2} > P^2_{1\pm}. \qquad (5)$$

On solving the system of equations for the characteristics of the kinetic equation in relativistic invariant form [6], we obtain a full set of first integrals for each plasma component

$$p_2^\circ = p_y^\circ = p_y + \varepsilon A/c = mcu_y + \varepsilon A/c = p_{y0},$$

$$p_3^\circ = p_z^\circ = mcu_z = p_{z0},$$

$$p_4^\circ = ic^{-1}(E^\circ + \varepsilon\Phi) =$$

$$= ic^{-1}(c\sqrt{m^2c^2 + p_x^2 + (p_y^\circ - \varepsilon A/c)^2} +$$

$$+ \varepsilon\Phi) = p_{40}. \qquad (6)$$

It is not difficult to see that relations (6) express the conditions for the conservation of the energy and spatial components in the generalized four-momentum of the particle. Curves for the spatial distribution of the basic physical quantities in the boundary layer for $\mu = m_-/m_+ = 1/1836$ are given in Figs. 2 and 3 for the values of the maximum dimensionless momentum $P_1 = 10^{-3}$ and $P_1 = 1$, respectively (see the corresponding curves for $\mu = 1/4$ in [1]). The quantity $\xi_* = x/\xi = (mc^2/4\pi n_0 e^2)^{1/2}$ has been taken as the unit of length. The symbols E, H, p, j, and $v_\pm$ denote, respectively, the following dimensionless quantities: the electrostatic and magnetic field strengths, the charge density and total current density, and the average velocity of the ion (electron component) in the direction of the y-axis. Dimensional quantities (in brackets) are associated with the dimensionless quantitites by the relations:

$$[E] = 10^{-3} m_- c^2 E / e\xi_*, \qquad [H] = 3.5\cdot 10^{-1} m_- c^2 H / e\xi_*,$$

$$[\rho] = 5\cdot 10^{-2} e n_0 \rho, \quad [j] = 2\cdot 10^{-2} ecn_0 j, \quad [v_-] = 10^{-1} cv_-;$$

$$[E] = 10 m_- c^2 E / e\xi_*, \quad [H] = 31 m_- c^2 H / e\xi_*, \quad [\rho] = en_0\rho,$$

$$[j] = ecn_0 j, \quad [v_\pm] = cv_\pm.$$

The basic physical characteristics of the boundary layer are shown in Figs. 4 and 5 as functions of energy. As in [1], the characteristic dimensions L_H, L_E, l, R, r, and D (the analog of the Debye radius) are determined for Fig. 4 by the formulas

$$H(-L_H) = 0.1 H_0, \quad E(-L_E) = 0.1 E_{\max}, \quad \rho(-l) = 0,$$

$$D^2 = \mu^{-1}[(1 + P_{1+}^2)^{1/2} - 1] + (1 + P_{1-}^2)^{1/2} - 1,$$

$$R = m_+ c^2 P_1 / eH_0, \quad r = m_- c^2 P_1 / eH_0,$$

and in Fig. 5,

$$H_0 = \frac{e\xi_*[H_0]}{m_- c^2}, \quad E_{\max} = \frac{e\xi_*[E_{\max}]}{m_- c^2}, \quad \rho_+, \rho_-,$$

$$j_- = \frac{[j_-]}{ecn_0}, \quad j = j_+ + j_-, \quad \psi_{\max}\frac{e[\Phi_{\max}]}{m_- c^2},$$

denote the maximum values of the corresponding quantities, and $\psi^\circ_{\max}$ is the dimensionless value of the electrostatic potential obtained on the assumption that the plasma is strictly neutral.

Numerical calculations carried out for the plasma model (5) enable us to draw some quantitative conclusions concerning the part played by charge separation in the redistribution of energy between components. The relativistic invariant form in which the problem is solved makes it possible to investigate this question over a wide range of plasma-particle kinetic energies.

Let us first consider one important energy relation. In accordance with (6) each plasma particle retains its total energy, i.e.,

$$E_\pm^\circ + \varepsilon_\pm\Phi = E_{0\pm}^\circ.$$

Here $E^\circ_{0\pm}$ is the energy as $x \to -\infty$; E° is to be understood as the "kinetic" energy

$$E^\circ = c\sqrt{m^2c^2 + p_x^2 + p_y^2}$$

which includes the potential energy.

Let us write an expression for the total energy of all the particles whose trajectories occupy the phase space xp. We have

$$\sum_\pm \int_{-\infty}^{\infty} \langle E_{0\pm}^\circ\rangle\, dx = \sum_\pm \int_{-\infty}^{\infty} dx \int_{G_\pm} [(E^\circ_\pm) + \varepsilon_\pm\Phi] f_\pm dp =$$

$$= \sum_\pm \int_{-\infty}^{\infty} \langle (E_\pm^\circ)\rangle\, dx + \int_{-\infty}^{\infty} \Phi(x)\langle\rho(x)\rangle\, dx \qquad (7)$$

The region $G_\pm$ in momentum space is determined from the conditions

$$p_{\pm}^2 \geqslant 0, \quad p_{0\pm}^2 \geqslant 0 .$$

Using (2) and (4), we transform the second integral in expression (7)

$$\int_{-\infty}^{\infty} \Phi \langle \rho \rangle \, dx = -\frac{1}{4\pi} \int_{-\infty}^{\infty} \Phi \frac{d^2\Phi}{dx^2} dx = -\frac{1}{4\pi} \Phi \frac{d\Phi}{dx} \Big|_{-\infty}^{\infty} +$$

$$+ \frac{1}{4\pi} \int_{-\infty}^{\infty} \left(\frac{d\Phi}{dx} \right)^2 dx = \int_{-\infty}^{\infty} \frac{E^2}{4\pi} dx . \qquad (8)$$

Combining (7) and (8), we have

$$\sum_{\pm} \int_{-\infty}^{\infty} \langle E_{0\pm}{}^{\circ} \rangle \, dx = \sum_{\pm} \int_{-\infty}^{\infty} \langle E_{\pm}{}^{\circ} \rangle + \int_{-\infty}^{\infty} \frac{E^2}{4\pi} dx , \qquad (9)$$

i.e., the total energy of all the particles which is "stored" as $x \to -\infty$ decays into the kinetic energy of the plasma and the energy of the electrostatic field which arises with charge separation.

Clearly if $\rho \equiv 0$ ("neutral approximation"), instead of exact relation (9) we have the expression

$$\sum_{\pm} \langle E_{0\pm}^{\bullet} \rangle = \sum_{\pm} \langle E_{\pm}^{\bullet} \rangle ,$$

which corresponds to formula (1).

Of course this expression can be made more exact if we take into account the effect of plasma polarization. We shall do this for plasma model (5). For the sake of clarity let us consider the simplest case $P_{1+} = P_{1-} = P_1$, in which electrons and ions have the same velocity distributions. This case corresponds to the Chapman-Ferraro model, if we give the particles a monoenergetic flux with (following [1] we call these "test particles")

$$p_{x0} = mcP_1, \qquad P_1 = \frac{U}{c} \left(1 - \frac{U^2}{c^2} \right)^{-1/2} .$$

In accordance with (6) we have

$$p_{y(-)} - eA_{(-)}/c = 0, \quad p_{y(+)} + eA_{(+)}/c = 0 ,$$

$$c\sqrt{m_-^2c^2 + p_y^2{}_{(-)}} - e\Phi_{(-)} = m_-c^2 \sqrt{1 + P_1^2} = E_{0-}{}^{\circ},$$

$$c\sqrt{m_+^2c^2 + p_{y(+)}} + e\Phi_{(+)} = m_+c^2 \sqrt{1 + P_1^2} = E_{0+}{}^{\circ}. \qquad (10)$$

In the neutral approximation

$$A_{(-)} = A_{(+)}, \qquad \Phi_{(-)} = \Phi_{(+)} = \Phi^{\circ}_{\max} ,$$

and for the relativistic analog of formula (1) we obtain

$$c\sqrt{m_-^2c^2 + p_{y(-)}^2} = E^{\circ}_{0-} + e\Phi^{\circ}_{\max} , \qquad (11)$$

where

$$e\Phi_{\max}{}^{\circ} = m_-c^2\psi^{\circ}_{\max} =$$

$$= m_-c^2 \left[\left(1 + \frac{P_1}{4\mu^2} \frac{4\mu + P_1(1+\mu)^2}{1 + P_1^2} \right)^{1/2} - (1 + P_1^2)^{1/2} \right] \qquad (12)$$

is determined from system (10).

Under real conditions the electrons are slightly separated from the ions and are reflected for some $\Phi = \Phi_{(-)} < \Phi^{\circ}_{\max}$, so that

$$c\sqrt{m_-c^2 + p_{y(-)}^2} = E_{0-}{}^{\circ} + e\Phi_{(-)} . \qquad (13)$$

Comparing (11) and (13) we see that when the separation of charges is taken into account, the trial electron, in comparison with the neutral approximation, does not receive its "full quota" of energy $e(\Phi^{\circ}_{\max} - \Phi_{(-)})$ from the ion, or in the relative expression $(\Phi^{\circ}_{\max} - \Phi_{(-)})/\Phi^{\circ}_{\max} = \alpha$, $0 < \alpha < 1$.

The value of the potential $\Phi_{(-)}$ for which reflection of the test electrons occurs was determined from numerical computer calculations of boundary-layer structure for various values of the limit momentum P_1. The curve for the coefficient $\alpha(P_1)$ for $\mu = 1/1836$ is given in Fig. 6. In the nonrelativistic energy region $\alpha \sim P_1$ and in the relativistic region α tends to a constant, approximately equal to 0.54.

Finally, in place of (1) we obtain the following expression for the energy of a test electron at the reflection point

$$E_{(-)}{}^{\circ} = c\sqrt{m_-^2c^2 + p_{y(-)}^2} =$$

$$= E_{0-}{}^{\circ} + (1 - \alpha) e\Phi^{\circ}_{\max} , \qquad (14)$$

where $\Phi^{\circ}_{\max}$ is determined from formula (12).

We write the formula for $p_{y(-)}(P_1, \mu, \alpha)$—the momentum of the test electron at the reflection point. From (10), (12), and (14) we have

$$p_{y(-)} = m_-c \left\{ \left[\alpha \sqrt{1 + P_1^2} + \right. \right.$$

$$\left. \left. + (1 - \alpha) \left(1 + \frac{P_1^2}{4\mu^2} \frac{4\mu + P_1^2(1+\mu)^2}{1 + P_1^2} \right)^{1/2} \right]^2 - 1 \right\}^{1/2} . \qquad (15)$$

In the nonrelativistic case $P_1 \ll 1$, with accuracy to terms on the order of P_1 and neglecting the constant μ, small in comparison with unity, we obtain

$$p_{y(-)} \approx m_-cP_1 [\alpha + (1 - \alpha)/\mu]^{1/2} .$$

In the limit case $P_1 \gg 1$

$$p_{y(-)} \approx m_-cP_1 [\alpha + (1 - \alpha)/2\mu] .$$

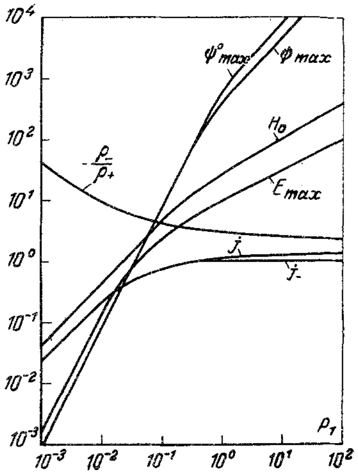
We recall that $p_{y(-)}$ is associated with $v_{(-)}$ by the formula

$$p_{y(-)} = m_-v_{(-)} (1 - v_{(-)}^2/c^2)^{-1/2} .$$

The dot-dash lines in Figs. 2 and 3 represent the spatial behavior of the quantities

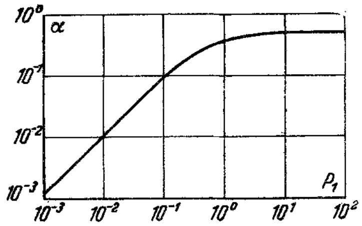
$$v_{\pm} = [v_{\pm}]/c = \langle j_{\pm} \rangle / \langle \rho_{\pm} \rangle c ,$$

the average velocities of the ion and electron components in a plane parallel to the boundary layer. The curves presented clearly illustrate the relation $v_{(-)}/v_{(+)} \approx 1/\mu$ in the nonrelativistic energy region and its violation for large P_1, when polarization effects play an important part. In the relativistic case (Fig. 3) a characteristic plateau is clearly seen in the curve for $v_{(-)}$, which corresponds to a stream of electrons having a velocity close to the velocity of light in the direction of the y-axis.



10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-3
10^-2
10^-1
10^0
10^1
10^2
ψ°max
φ max
H0
-ρ-/ρ+
Emax
j+
j-
ρ1

Fig. 5



10^0
α
10^-1
10^-2
10^-3
10^-3
10^-2
10^-1
10^0
10^1
10^2
ρ1

Fig. 6

REFERENCES

1. Yu. S. Sigov, "The structure of a boundary layer between a rarefied plasma and a magnetic field," PMTF Journal of Applied Mechanics and Technical Physics], no. 2, 1965.

2. V. C. A. Ferraro, Theory of a plane model, J. Geophys. Res., vol. 57, no. 1, 1952.

3. V. I. Veksler, "A new mechanism for the generation of relativistic electrons in cosmic space," Dokl. AN SSSR, vol. 118, no. 2, 1958.

4. Yu. S. Sigov, "Electrostatic effects in the motion of a rarefield plasma in a nonuniform magnetic field," Dokl. AN SSSR, vol. 138, no. 6, 1961.

5. V. P. Shabanskii, "The structure of the transition layer between a plasma and a magnetic field," ZhETF, vol. 40, no. 5, 1961.

6. S. T. Belyaev and G. I. Budker, "The relativistic kinetic equation," Dokl. AN SSSR, vol. 7, no. 6, 1956.

21 July 1966 Moscow